Hybrid exotic meson with $J^{PC} = 1^{-+}$ in AdS/QCD

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ABSTRACT: We investigate the hybrid exotic meson with $J^{\mathrm{PC}} = 1^{-+}$ within the framework of an AdS/QCD model. Introducing a holographic field dual to the operator for hybrid exotic meson, we obtain the eigen-value equation for its mass. Fixing all free parameters by AdS/CFT dictionaries, we predict the masses of the hybrid exotic meson. The results turn out to be 1476 MeV for the ground state, and 2611 MeV for the first excited one. Being compared with the existing experimental data for the $\pi_1(1400)$, which is known to be $m_{\pi_1} = 1351 \pm 30$ MeV, the present result seems to be qualitative in agreement with it. We also predict the decay constant of $\pi_1(1400)$: $F_{\pi_1} = 10.6$ MeV.

Contents

1. Introduction

There has been a considerable amount of interest in exotic mesons well over decades, since it cannot be explained by the conventional constituent quark model in which mesons consist of quark and antiquark pairs ($q\bar{q}$). The nonexotic mesons are restricted to have quantum numbers constrained by the following selection rules: $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, where L and S denote the relative orbital angular momentum and total spin of quarks consisting of mesons. Thus, an exotic meson with $J^{\mathrm{PC}} = 1^{-+}$ cannot be explained as a $q\bar{q}$ state. It is only possible to produce such a state either as a multi-quark state (tetraquark) or as a quark-gluon hybrid state. In particular, it is of great importance to understand the quark-gluon hybrid exotic mesons, since it provides a key to examine the role of the gluons as basic building blocks for hadrons.

Since Jaffe and Johnson suggested a possible existence of hybrid mesons [1], there has been a great deal of theoretical investigations (see a recent review [2] for full references): For example, the bag model [3–7], the flux tube model [8–12], the QCD sum rules [13–18], lattice QCD [19–21], etc. Deep electro-production of exotic hybrid mesons was studied in [22]. Experimentally, a hint of the exotic meson was already observed many years ago [23]. Later, the lowest-lying hybrid exotic meson has been found by various experimental collaborations [24–29] and was christened $\pi_1(1400)$. The $\pi_1(1400)$ is now announced with its mass $m_{\pi_1} = 1351 \pm 30\,\mathrm{MeV}$ and width $\Gamma_{\pi_1} = 313 \pm 40\,\mathrm{MeV}$ by the Particle Data Group (PDG) [30].

The AdS/CFT correspondence [31–33] that connects a strongly coupled large N_c gauge theory to a weakly coupled supergravity provides novel and attractive insight into nonperturbative features of quantum chromodynamics (QCD) such as the quark confinement and spontaneous breakdown of chiral symmetry (SBχS). Though there is still no rigorous theoretical ground for such a correspondence in real QCD, this new idea has triggered a great amount of theoretical works on possible mappings from nonperturbative QCD to $5D$ gravity, i.e. holographic dual of QCD. In fact, there are in general two different routes to modeling holographic dual of QCD (See, for example, a recent review [34]): One

way is to construct 10 dimensional (10D) models based on string theory of D3/D7, D4/D6 or D4/D8 branes [35 – 39]. The other way is so-called a bottom-up approach to a holographic model of QCD, often called as AdS (Anti-de Sitter Space)/QCD [40 – 42] in which a 5D holographic dual is constructed from QCD, the 5D gauge coupling being identified by matching the two-point vector correlation functions. Despite the fact that this bottom-up approach is somewhat on an ad hoc basis, it reflects some of most important features of gauge/gravity dual. Moreover, it is rather successful in describing properties of hadrons (See the recent review [34]).

In the present work, we want to investigate the quark-gluon hybrid meson with $J^{\mathrm{PC}} = 1^{-+}$, in particular, $\pi_1(1400)$, based on the AdS/QCD model developed by refs. [40 – 42]. Since the hybrid exotic mesons exist in large N_c QCD as narrow resonant states [43], it is worth while to study it within AdS/QCD. Moreover, the AdS/QCD model has a virtue in dealing with gluonic operators for ease of application, since the 5D bulk fields corresponding to them can be much more easily handled. In AdS/QCD, the $\pi_1(1400)$ may be regarded as a spin-1 bulk field with quantum number $J^{\mathrm{PC}} = 1^{-+}$. It can be identified as the ground state of the Kaluza-Klein (KK) modes in this channel. The eigenvalue equation for the $\pi_1(1400)$ is derived from the classical equation of motion from the 5D action, so that the mass of the $\pi_1(1400)$ can be obtained by solving this equation. In fact, we will show that the $\pi_1(1400)$ mass arises from the first zero of the modified Bessel function K_3.

The present work is organized as follows: In section 2, we briefly review the present status of theoretical works for the hybrid exotic mesons. In section 3, we explain briefly the hard-wall AdS/QCD model with the bulk field for the hybrid exotic meson taken into account. Then, we present the result for the mass of the hybrid exotic meson with $J^{\mathrm{PC}} = 1^{-+}$ and compare it with those of various models and experimental data. We also show the result of the decay constant of the π_1 hybrid exotic meson. In the last section, we summarize the present work.

2. Hybrid exotic meson with $J^{\mathrm{PC}} = 1^{-+}$

As already mentioned in Introduction, the hybrid exotic mesons have been studied extensively in many different theoretical frameworks. In 1980s, various versions of the MIT bag model with transverse gluon fields were invoked to predict the existence of the hybrid exotic meson with $J^{\mathrm{PC}} = 1^{-+}$ under the name of $q\bar{q}G$ hermaphrodite meson or meikton [3 – 7]. In the MIT bag model, the lowest state of hybrid exotic mesons consists of $(q\bar{q})$ and of a transverse TE (magnetic) gluon that is the lowest gluonic eigenmode due to the MIT boundary conditions. Having considered $\mathcal{O}(\alpha_{\mathrm{s}})$-order energy shifts, refs. [6, 7] predicted the mass of the lowest hybrid exotic state with $J^{\mathrm{PC}} = 1^{-+}$ to be around 1400 MeV.

The flux tube model was frequently used to investigate the hybrid exotic mesons. The model was extracted from the strong coupling Hamiltonian lattice formulation of QCD [44]. The term, "flux tube", mimics a roughly cylindrical region of chaotic gluon fields, which confines widely separated static color sources. This flux tube leads to a confining linear potential between color singlet q and $\bar{q}$. The model contains normal modes of excitation

only to the locally transverse spatial direction. The model predicts the lowest state of hybrid exotic meson in the range of $1800 - 2100\,\mathrm{MeV}$ [8 – 11, 45, 12].

In QCD sum rules, the predictions of the mass of the hybrid exotic meson with $J^{\mathrm{PC}} = 1^{-+}$ do not seem consistent: For example, Balitsky et al. [13, 14] estimate the mass around $1000 - 1300\,\mathrm{MeV}$, while Latorre et al. [15] vote for $M(1^{-+}) \approx 2.1\,\mathrm{GeV}$. Refs. [16 – 18] suggest even $M(1^{-+}) \approx 2500\,\mathrm{MeV}$.

While lattice QCD is the most promising way of describing low-energy phenomena in QCD, it is still far from the real world, since the pion mass in lattice QCD is still larger than the physical pion mass $m_\pi = 139.57\,\mathrm{MeV}$. In fact, Thomas and Szczepaniak [47] examined chiral extrapolations in exotic meson spectrum and found that the linear extrapolation does not seem working, since the self-energy corrections to the exotic meson mass are most likely to introduce some non-linearity in the chiral extrapolation of lattice calculation of its mass.

In QCD, the hybrid exotic meson with $J^{\mathrm{PC}} = 1^{-+}$ can be treated as a vector operator consisting of the quark, antiquark and gluon:

$$J_\mu^a(x) \;=\; \bar{\psi}(x)T^a G_{\mu\alpha}(x)\gamma^\alpha \psi(x), \tag{2.1}$$

where $\psi(x)$ and $G_{\mu\alpha}$ denote the quark field and the gluon field strength defined as $G_{\mu\alpha} = G_{\mu\alpha}^A t^A$ with color matrices t^A ($\mathrm{tr}[t^A t^B] = \delta_{AB}/2$). The T^a represent the flavor matrices and we take it as Pauli matrices, since we consider only flavor SU(2) in the present work. The two-point correlation function [14] for the vector current in eq. (2.1) is written as

$$\Pi_{\mu\nu} \;=\; \int d^4 x e^{iq\cdot x}\langle T(J_\mu^a(x)J_\nu^b(0))\rangle_0 \;=\; -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_V^{ab}(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_S^{ab}(q^2), \quad (2.2)$$

where $\Pi_V(q^2)$ includes the intermediate hybrid exotic vector mesons with $J^{\mathrm{PC}} = 1^{-+}$, whereas $\Pi_S(q^2)$ contains the hybrid exotic scalar mesons $J^{\mathrm{PC}} = 0^{++}$. The result of the operator product expansion (OPE) [14] is given as follows:

$$\Pi_V(q^2) = -\frac{(N_c^2 - 1)}{2^7 15\pi^3}q^6 \ln(-q^2) + \cdots, \tag{2.3}$$

$$\Pi_S(q^2) = -\frac{(N_c^2 - 1)}{2^8 15\pi^3}q^6 \ln(-q^2) + \cdots, \tag{2.4}$$

where $\Pi_{V,S}\delta^{ab}/2 = \Pi_{V,S}^{ab}$.

We will use this expression (2.4) to fix the 5D bulk field for the hybrid exotic meson.

3. Hybrid exotic mesons in AdS/QCD

The metric of an AdS space is defined as

$$ds^2 \;=\; g_{MN}dx^M dx^N \;=\; \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \tag{3.1}$$

where $\eta_{\mu\nu}$ denotes the 4D Minkowski metric: $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The AdS space is compactified: IR boundary at $z = z_m$, and UV at $z = \epsilon \to 0$. Thus, the model is justified

within the range: $\epsilon \leq z \leq z_m$. The 5D gauge action in AdS$_5$ space with the bulk field for the hybrid exotic meson with $J^{\text{PC}} = 1^{-+}$ contained can be expressed as

$$S = \int d^4x dz \sqrt{g} \, \text{Tr} \left[-\frac{1}{4\tilde{g}_5^2} F_{MN} F^{MN} + \frac{1}{2} m_5^2 V_M V^M \right], \tag{3.2}$$

where $F_{MN} = \partial_M V_N - \partial_N V_M - i[V_M, V_N]$ and $M, N = 0, 1, 2, 3, 4$. The gauge field, which is dual to the 4D current in eq. (2.1), is defined as $V_M = V_M^a t^a$ with $\text{tr}(t^a t^b) = \delta^{ab}/2$. Note that the 5D gauge coupling $\tilde{g}_5$ is not identical to the gauge coupling in [40, 41], where the bulk gauge field is dual to a 4D vector current $\bar{\psi}(x)\gamma^\mu t^a \psi(x)$. The 5D mass m_5^2 of the bulk field V_M is determined by the relation $m_5^2 = (\Delta - p)(\Delta + p - 4)$ [32, 33], where Δ stands for the dimension of the corresponding operator with spin p. Since the dimension and spin of the operator in eq. (2.1) are $\Delta = 5$ and $p = 1$, respectively, we get the 5D mass of the bulk field V_M is: $m_5^2 = (5-1)(5+1-4) = 8$. The hybrid exotic meson $\pi_1(1400)$ may be regarded as the ground state of the Kaluza-Klein (KK) modes. We remark that a scalar condensate of $q\bar{q}G$, quark-gluon mixed condensate, was studied in an AdS/QCD model [46].

In ref. [48], it was shown that for small z or near the boundary of AdS space, a 5D field $\phi(x, z)$ dual to a 4D operator $\mathcal{O}$ can be expressed as

$$\phi(x, z) = z^{4-\Delta-p} \left[\phi_0(x) + O(z^2) \right] + z^{\Delta-p} \left[A(x) + O(z^2) \right], \tag{3.3}$$

where $\phi_0(x)$ is a prescribed source function for $\mathcal{O}(x)$ and $A(x)$ denotes a physical fluctuation that can be determined from the source by solving the classical equation of motion. It can be directly related to the vacuum expectation value (VEV) of the $\mathcal{O}(x)$ as follows [48]:

$$A(x) = \frac{1}{2\Delta - 4} \langle \mathcal{O}(x) \rangle. \tag{3.4}$$

Therefore, the bulk field V for the exotic meson has the following asymptotic form at the boundary $z = \epsilon$:

$$V(z) = c_1 \frac{1}{z^2} + c_2 z^4, \tag{3.5}$$

where c_1 is the source term.

We now fix the 5D gauge coupling $\tilde{g}_5$ by matching the two-point vector correlation function obtained with the action in (3.2) to the leading contribution from the OPE result shown in eq. (2.3) [14]. Here we choose the axial-like gauge condition $V_z(x, z) = 0$. It can be decomposed into the transverse and longitudinal parts: $V_\mu = (V_\mu)_\perp + (V_\mu)_\parallel$. Using the Fourier transform of the vector field: $V_\mu^a = \int d^4x e^{iq \cdot x} V_\mu^a(x, z)$, we can write the equation of motion for the transverse part of the vector field as follows:

$$\left[\partial_z \left(\frac{1}{z} \partial_z V_\mu^a(q, z) \right) + \left(\frac{q^2}{z} - C_5^2/z^3 \right) V_\mu^a(q, z) \right]_\perp = 0, \tag{3.6}$$

where $C_5^2 \equiv m_5^2 \tilde{g}_5^2$. The corresponding solution of eq. (3.6) can be expressed as a separable form:

$$(V_\mu^a(q, z))_\perp = V(q, z) \overline{V}_\mu^a(q), \tag{3.7}$$

where $\overline{V}_\mu^a(q)$ is the Fourier transform of the source of the 4D vector-current operator $\bar{\psi}T^a G_{\mu\alpha}\gamma^\alpha\psi$. The explicit solution for $V(q,z)$ can be derived by solving eq. (3.6),

$$V(Q,z) = c_1 z I_n(Qz) + c_2 z K_n(Qz),$$ (3.8)

where I_n and K_n are modified Bessel functions, $n = \sqrt{1 + C_5^2}$, and $Q^2 = -q^2$. The asymptotic form of the bulk vector field at the boundary $z = \epsilon$, which is shown in eq. (3.5), dictates the following boundary condition: $V(Q,\epsilon) = c/\epsilon^2$. Here, c is a constant to be fixed. We refer to ref. [42] for a similar procedure for a non-exotic scalar two-point correlator. Imposing the UV boundary condition $V(Q,\epsilon) \sim 1/\epsilon^2$, we obtain $n = 3$ ($C_5^2 = 8$), and $c_2 = cK_3^{-1}(Q\epsilon)/\epsilon^3$. Note that $C_5^2 = 8$ implies $\tilde{g}_5^2 = 1$ since $m_5^2 = 8$. Thus, the asymptotic behavior of the bulk vector field dictated by the AdS/CFT correspondence uniquely determines the 5D gauge coupling. Since the two-point vector correlation function will be obtained at the UV, $z = \epsilon$, we will not explicitly consider the IR boundary condition that will fix c_1. Then, following the standard procedure given in [40–42], we obtain

$$\Pi_V(Q^2) = -\frac{1}{64}c^2 Q^6 \ln Q^2.$$ (3.9)

Comparing this with the OPE given in eq. (2.3) in the leading Q^2 order, we obtain

$$c^2 = \frac{N_c^2 - 1}{30\pi^3}.$$ (3.10)

We are now in a position to predict the mass of $\pi_1(1400)$. Note that we have no free parameter that we may play around to get a desirable value of m_{π_1}. The mode equation for the exotic meson reads

$$\left(\partial_z^2 - \frac{1}{z}\partial_z + m_i^2 - \frac{8}{z^2}\right)f_i(z) = 0,$$ (3.11)

where $V_\mu(x,z) = \sum_i f_i(z)V_\mu^{(i)}(x)$. The solution is given by, in terms of the Bessel functions,

$$f_i(z) = a_1 z J_3(m_i z) + a_2 z Y_3(m_i z).$$ (3.12)

The KK masses for the exotic mesons are fixed by the IR boundary condition $\partial_z f_i(z)|_{z=z_m} = 0$:

$$m_i z_m J_2(m_i z_m) - 2J_3(m_i z_m) = 0.$$ (3.13)

Taking $i = 1$ for the ground state, we obtain the mass of π_1 to be $m_{\pi_1} \simeq 1476\,\text{MeV}$ that is close to the experimental value $m_{\pi_1} = 1351 \pm 30\,\text{MeV}$ [30]. In table 1, we compare the present result with other models. The next excited mass for the hybrid exotic meson turns out to be $2611\,\text{MeV}$. As usual, the excited mass seems to be quite large, compared to the ground state.

Finally, we consider decay constant of π_1. The decay constant of the π_1 [14] is defined as

$$\langle 0|\bar{\psi}(x)T^a G_{\mu\alpha}(x)\gamma^\alpha\psi(x)|\pi_1^b(p)\rangle = \frac{1}{\sqrt{2}}F_{\pi_1}m_{\pi_1}^3\varepsilon_\mu\delta^{ab}e^{ip\cdot x},$$ (3.14)

Models	Mass [MeV]
Present model	1476
Flux tube models [8 – 11, 45, 12]	1800–2100
Bag models [6, 7]	1300–1800
QCD sum rules [13 – 18]	1200–2500
Lattice QCD [19 – 21]	1800–2300
Experiment (PDG) [30]	1351 ± 30

Table 1: Comparison with the results of other models

where F_{π_1} denotes the decay constant of the π_1. The ε_μ is for its polarization vector. Following the procedure sketched in [40], we arrive at the definition of F_{π_1} in AdS/QCD:

$$(F_{\pi_1} m_{\pi_1}^3)^2 = \frac{c^2}{\tilde{g}_5^2} \left(\frac{f_1'(\epsilon)}{\epsilon^3} \right)^2 , \qquad (3.15)$$

where $f_1(z)$ is the normalized solution of the mode equation, eq. (3.11), with $i = 1$. Note that the definition of $F_{\pi_1}^2$ is slightly different from that of ρ-meson given in [40]:

$$F_\rho^2 = \frac{1}{g_5^2}[\psi_\rho'(\epsilon)/\epsilon]^2 . \qquad (3.16)$$

The discrepancy is basically due to a difference in UV boundary conditions: $V(q, \epsilon) = c/\epsilon^2$ for $\pi_1(1400)$, $V(q, \epsilon) = 1$ for ρ-meson. Taking $N_c = 3$ in eq. (3.10), we obtain $F_{\pi_1} = 10.6 \,\mathrm{MeV}$. Compared to the the value of the decay constant from the QCD sum rules [14] $F_{\pi_1} \approx 20 \,\mathrm{MeV}$, the result is qualitatively in almost the same order.

4. Summary

The present work has aimed at investigating the hybrid exotic mesons with $J^{\mathrm{PC}} = 1^{-+}$ within the framework of AdS/QCD. We have first introduced the 5D bulk field dual to the 4D quark-gluon vector current $\bar{\psi}T^a G_{\mu\alpha}\gamma^\alpha\psi$ and constructed the 5D bulk action for the exotic meson. Solving the classical equation of motion for the transverse part of the hybrid exotic vector field, we have obtained the explicit form of the vector field in terms of the modified Bessel functions with index $n = \sqrt{1 + m_5^2 \tilde{g}_5^2}$. Imposing the UV boundary condition for the vector field to calculate the two-point correlation function, we have determined $n = 3$, which fixes the 5D gauge coupling. We have then obtained the eigenvalue equation for the hybrid exotic meson mass.

In order to find the mass of the hybrid exotic meson $\pi_1(1400)$, we have identified it as the ground state of the KK modes. The mass of the $\pi_1(1400)$ turned out to be $m_{\pi_1} \simeq 1476 \,\mathrm{MeV}$ that is close to the experimental data: $1351 \pm 30 \,\mathrm{MeV}$, which is a remarkable result, considering the fact that the formalism from AdS/QCD is so simple. The mass of the next excited state in the hybrid exotic channel turns out to be $2611 \,\mathrm{MeV}$. Similar to the ρ mesons in the hard wall model for non-exotic mesons [40, 41], the mass of the excited state seems quite large in the present study. We also predicted the decay constant of π_1 (1400): $F_{\pi_1} = 10.6 \,\mathrm{MeV}$. Finally, we remark that it will be interesting to see if one can study the hybrid exotic meson in a stringy set-up.

Acknowledgments

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